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Iterative mesh segmentation using approximated Voronoi Diagram

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Abstract

Mesh segmentation and partitioning of 3D models have always been significant as one of the most structural tools used in many applications of CAD and computer graphics. One of the most versatile of these algorithms, which is capable of optimum segmentation of model, is the iterative algorithm. It segments the model in an optimized way based on Lloyd algorithm, and by forming Voronoi diagram through points cloud data. The most remarkable disadvantage of iterative algorithms is their long solving time which is caused by iteration of algorithm in order to yield the best segmentation or, in other words, the best Voronoi diagram on the model. In this paper, using an approximation of Voronoi diagram, a method has been presented to obtain the optimum segmentation in a shorter time relative to other iterative algorithms.

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Keywords: mesh segmentation; Lloyd clustering; geometric approximation; approximate Voronoi Diagram; iterative clustering

1. Introduction

Mesh segmentation or partitioning has turned into a fundamental element in many algorithms used in geometrical modeling and computer graphics. Mesh classification is used in several applications such as parameterization [1], texture mapping [2], shape matching [3], morphing [3], multi resolution modeling [4], data compression [5], mesh correction, collision detection [6], skeleton extraction [7], and picture animation [5].

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Segmentation of meshed model, or more generally, segmentation of geometrical shapes can be classified in terms of the aspect of view to the initial model, methodology, the application of output data, and geometrical standard of measurement for allocating elements to suitable regions.

Since no unique standard is used for the segmentation of 3D models, to compare the output of different methods, the classification problem is considered as an optimization problem. To define the optimization problem for determining mesh's grid energy, a measurement standard is used, on which the partitioning is based. Each partitioning which causes more decrease in the mesh's grid energy would be a more suitable one.

Iterative algorithm which is a parametric method, tends to decrease mesh's grid energy in each iteration, thereby yielding an optimum answer.

Shlafman has used the iterative segmentation algorithms to partitioning of separate models for convenience in calculation of homomorphism mapping among different, but with-identical-topology, models [8].

Steiner simultaneously carried out the optimum segmentation and reconstruction of model using flat planes [13]. The optimum answer is a condition when the model is reconstructed with the least number of planes as well as the least approximation value. The indicator element of each region is a flat plane used to approximate and reconstruct the model. To decrease the approximation value, two error measurement standards have been used. The first standard is based on Euclidean distance norm (L2), and the second one is L2,1 norm which has better functionality compared with L2 norm in terms of both simplicity and obtained results. L2,1 norm is calculated over normal vectors' field of faces, using Euclidean norm formulation. Since normal vectors are the base of visual and rendering systems, it is expected that the partitioned regions in this method show better appearance results in comparison with the latter. Although the presented method in [9] is very efficient, since the model is approximated solely by flat planes, undesirable results are often obtained in classification of complicated shapes and in cases that partitioned regions are required to be confined to a possible extent. Hence, Wu segmented the model utilizing the advantages of iterative classification algorithms and choosing more complicated surfaces such as sphere, cylinder, and roll as approximating elements [10].

Julius segmented the model into extendable regions, i.e. regions with zero Gaussian deviation, using iterative algorithm [11]. Machado segmented the model into extendable regions and surfaces without consideration of curvature, using Lambert model [12]. To increase the algorithm's efficiency and in order to find optimum indicator elements, the Voronoi diagram has been used. In Voronoi diagram it is required that the center of regions be updated in each iteration so as to obtain the best position. Inasmuch as the regions are continuous and the changes regarding the center of each region must fall within the manifold surface, Lloyd algorithm, i.e. iterative method, has been employed to yield the best basic elements.

Wu used a new type of iterative algorithm to classify and label the model [13]. In this algorithm, the indicator elements are regarded as a set of labels which in addition to acquiring the new set's properties during each iteration, they preserve some properties of the previous set as well.

In spite of their high precision, iterative algorithms have constraints due to unsuitable solving time. The reason for long segmentation time lies in the way the partitioning problem is solved. As mentioned, iterative algorithms use Lloyd algorithm to solve a partitioning problem. Thus, the Lloyd algorithm is considered in next section.

2. Lloyd algorithm

Classification of a set of separate points mean their partitioning to separated regions, provided that the distance of points corresponding to a region from each other is less than their distance relative to the points of the other regions. Lloyd algorithm is a permanent method based on the iteration of a fixed point, which can generate such classification [9]. The general idea of Lloyd algorithm which is also recognized as iterative Voronoi algorithm, is very simple. In this method, each class is introduced by an indicator element which is usually the center of that class. First, k random centers are chosen out of input data space as indicator element. Then, the classification is carried out by allocating each data to the closest center. After that, the algorithm performs the segmentation again by updating the indicator elements and choosing the center of each region as the new indicator element. The trend iterates until the stop condition is provided.

Lloyd algorithm was first used for quantization in the field of PCM in electrical engineering and, due to its simplicity, was quickly adopted in other areas as well [14]. This method has less deviations compared to other ones which use particle spreading technic, and this is the reason for its extensive application in many fields such as graphics, dithering, stippling [15], finite elements for mesh smoothing [16], and quantization in various areas [17].

Lloyd algorithm is also used extensively to find the minimum point (which is not necessarily general) in functions of higher orders. In regard to segmentation of geometrical models, instead of compactness optimization for each region, Lloyd algorithm may be employed as a tool for reducing the deviation of each class from the indicator element of that class.

The goal of Lloyd algorithm which is called k-means method in some cases, is to minimize the grid's energy E based on a way in which different regions are best identified and classified. Grid's energy E with N points $\{\boldsymbol{X_j}\}$ and k centers $\{\boldsymbol{c_i}\}$ is defined as:

$$E = \sum_{i\in 1\ldots k} \sum_{X_j \in R_i} \left\| X_j - c_i \right\|^2 \tag{1}$$

The point that Lloyd algorithm and k-means method are different in terms of input data, is of high importance. The input of k-means algorithm is a set of separate points, whereas that of Lloyd algorithm is the regions with geometrical continuousness. Therefore, when the inputs are rearranged (classified again), instead of simply measuring the centre's distance from each point which occurs in k-means, Voronoi diagram is used in Lloyd algorithm.

2.1. Voronoi diagram

Assume that P is a set of n points on a plane $\epsilon\pi\mathbb{R}^2$. One of the most fundamental problems in a variety of applications is to obtain data structure on P in a way that can efficiently result the closest neighborhood. There are appropriate solutions to solve this problem in 2D space. The most general among them to find the closest point is to create Voronoi diagram on P (which is presented as vor(p)). Voronoi diagram is a basic calculational tool in geometrical computations, which is used in many applications such as clustering, motion planning, learning, and surface reconstruction [18].

The general idea is that an impact region is attributed to each point, provided the sum of the regions equal π plane. However, in higher dimensions, e.g. models used in CAD applications which are all 3D, it is not very easy to yield the closest point, i.e. impact region.

The Voronoi region of P point in a limited set $S \subset \mathbb{R}^3$ is a set of points very closer to P than other point S:

$$V_p = \left\{ x \in R^3 \mid \left\| x - p \right\| \le \left\| x - q \right\|, \forall q \in S \right\} \tag{2}$$

Notice that semi space of points are closer to p than q:

$$H_{pq} = \left\{ x \in R^3 \mid \left\| x - p \right\| \le \left\| x - q \right\| \right\} \tag{3}$$

If the Euclidean distance standard is applied, since the Voronoi region for p is created by semi spaces H_{pq} for all $q \in S - \{p\}$, a convex polyhedron V_p is generated. In segmentation of models used in CAD, it is very crucial to maintain the conditions of continuousness, hence the Euclidean measurement standard is not appropriate and the geometrical measurement should be used.

Although the generalization of Voronoi diagram for classification of cases other than the points is also very common, its usage would encounter difficulties in higher dimensions due to cumbersome calculations. Complexity function of calculations in $\mathbb{R}^d$ space to form Voronoi diagram equals $\theta(n^{[d]})$. So, the more the problem dimensions are, the more inapplicable the creation and saving of these diagrams in terms of calculations become.

In iterative algorithms, due to cumbersome calculations of Voronoi diagram in each iteration of algorithm, the solving trend is very slower relative to other segmentation algorithms. For this reason, regardless of previous works which were all using Voronoi diagram for segmentation, an approximation of Voronoi diagram is applied in this paper.

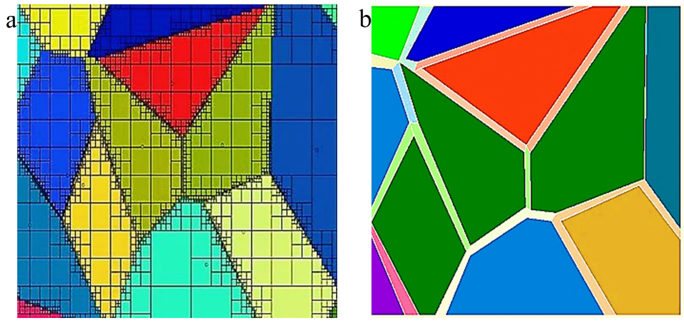

Fig. 1. (a) Approximate the Voronoi Diagram with a quadtree; (b) Voronoi cells with fuzzy boundary

2.2. Approximate Voronoi diagram

To reduce time and space required for creation of Voronoi diagram, Peled presented an approximation of Voronoi diagram V_p^* for $\mathbb{R}^d$ space in near linear time [19]. This algorithm aims to partition the $\mathbb{R}^d$ space, provided that if the indicator of each region c is $p_c \in S$, then, for all the points on region c, p_c point is approximately the closest point to them in P grid:

$$V_p^* = \left\{ x \in R^3 \mid \|x - p\| \leq (1+\varepsilon)\|x - q\|, \forall q \in S \right\} \tag{4}$$

0< ε is a constant value. According to Fig. 1, the boundary of partitioned regions in approximation of Voronoi diagram turns fuzzy. In this paper, the fuzzy region has been utilized to obtain the primary guess for appropriate distribution of central points and thus, reducing run time of iterative algorithm.

3. Use of Voronoi diagram approximation for segmentation

As described in iterative algorithms, the positions of central points and so the regions change by each iteration to yield the best segmentation. In Fig. 2 segmentation of a simple model using iterative algorithm is shown in initial and final stages (after 30 iterations).

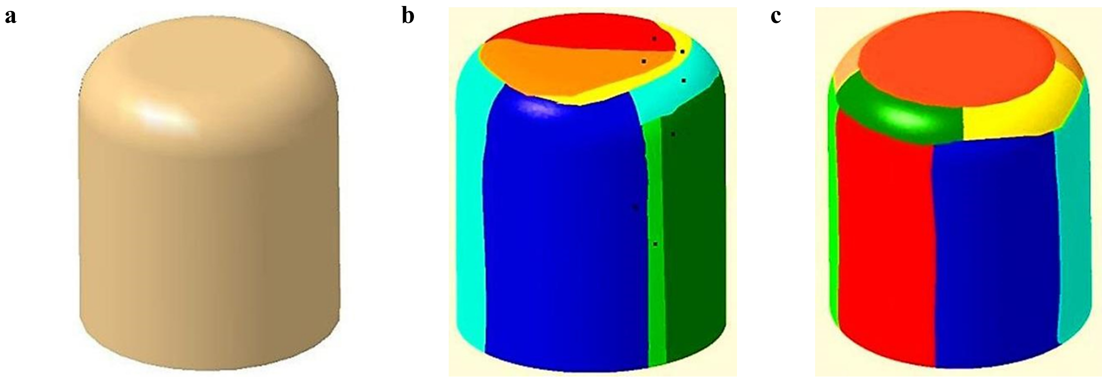

Fig. 2. (a) Triangulated model with 3KT (b) disterbution of initial central elements and segmentation model (c) final segmentation model

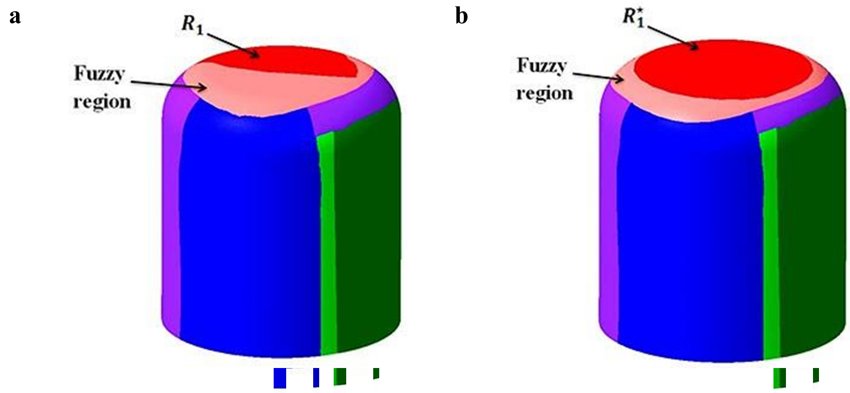

Fig. 3. (a) fuzzy region for R_1 (b) R_1 merged with two adjacent region and formed new R_1^*

For the proposed model, the approximation of Voronoi diagram and the calculation of fuzzy region between different regions is simply calculated using $\boldsymbol{L^{2,1}}$ norm. After the calculation of diagram, first the regions with more extensive fuzzy areas are investigated for pace increase. Demonstration of fuzzy regions for the whole model simultaneously causes appearance complexity and ambiguity. For better comprehension, fuzzy region for a given region is shown individually in Fig. 3. As seen in this figure, the fuzzy region covers some regions thoroughly, which means that these regions have the capabilities to combine and be integrated into a single unit.

Region merging: To investigate the likeliness for merging of two or more adjacent regions, a very simple condition has been applied: If the central element of a region is located on fuzzy area of another region, then the two regions are integrated and the central element of smaller region is removed from the $\{c_i\}$ set. In fact, the larger region surrounds the smaller region.

As seen in the figure, the fuzzy segmentation completely overlaps with the fillet surface. It must be noted that such use of fuzzy diagram is not considered appropriate and may cause errors, because although the approximated Voronoi diagram is more efficient than Voronoi diagram in showing and specifying different regions and sections of the model, but also causes over segmentation of the model, especially that we have to take $|\varepsilon|$ slightly greater than usual for better exhibition of fuzzy regions and for increasing the effectiveness and efficiency of the algorithm.

To prevent the over segmentation, upon obtaining the approximation of Voronoi diagram, dispersion analysis is carried out on fuzzy regions to determine the effective fuzzy regions. By 'effective fuzzy regions' we mean the regions which are able to form a new region without over segmentation of the model. With filtering the small fuzzy regions, the effective regions may be suitably identified after the first stage of segmentation.

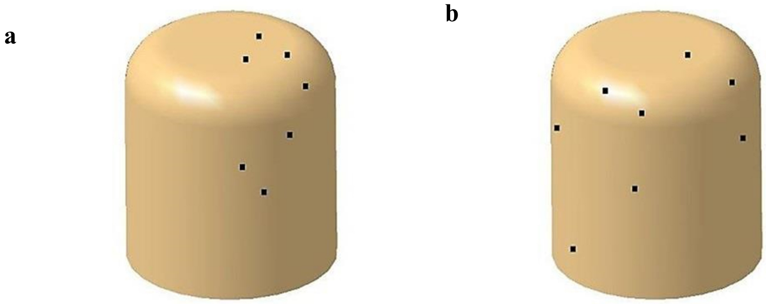

Fig. 4. (a) random distribution of initial central elements (b) redistribution of central elements according to the approximate voronoi diagram's of random ones

With identification of integrable regions as well as effective fuzzy regions, new central elements are distributed. In fact, the approximated Voronoi diagram is used as a guide for suitable distribution of central elements, Fig. 4. By correcting the position of central elements and by their redistribution according to approximated Voronoi diagram, the given model has been segmented again using iterative algorithm. As expected, segmentation reached the convergence so faster, merely by 8 iterations.

4. Conclusion

In this paper, using the approximation of Voronoi diagram, a very simple solution to appropriate distribution of initial elements for iterative algorithm has been proposed. As the Voronoi diagram itself and the classification, which is carried out at the first stage, have been used in this method to guess the position of initial elements, no particular calculational load is imposed on the algorithm. The use of proposed algorithm, and so achieving the appropriate distribution of central elements, causes the classification algorithms to reach the desirable convergence much faster.

Since the precision of this method in guessing the position of initial elements depends on the extent of fuzzy region, we are expecting to obtain the suitable value of $|\varepsilon|$ and run the algorithm for more complicated models in forthcoming works.

References

[1] Sheffer A, Praun E, Rose N. Mesh Parameterization Methods and Their Applications. J Computer Graphics and Vision 2006;2:105-71.
[2] Zhange E, Mischaikow K, Turk G. Feature based surface parameterization and texture mapping. ACM Transaction on Graphics 2005;24:1-27.
[3] Zuckerberger E, Tal A, Shlafman S. Polyhedral surface decomposition with applications. J Computers & Graphics 2002;26:733-43.
[4] Boier M. Domain decomposition for multiresolution analysis. 2th Eurographics Symposium on Geometry Processing 2003:29-40.
[5] Leet Y, Lin P, Yans U, Lin C. Mesh decomposition using motion information from animation sequence. J Computer Animation and Virtual Worlds 2005;16:519-29.
[6] Li X, Toon T, Tan T, Huang Z. Decomposing polygon meshes for interactive applications. In Proceedings of the symposium on Interactive 3D graphics 2001;35-42.
[7] Mattia N, Silvia B, Giuseppe P, Bianca F. Graph-based representations of point clouds. J Graphical Models 2011;73:151-64.
[8] Shlafman S, Tal A, Katz S. Metamorphosis of polyhedral surfaces using decomposition. J Computer Graphics forum 2002;21:219-28.
[9] Cohen Steiner D, Alliez P, Desbrun M. Variational shape approximation. ACM Transactions on Graphics 2004;23:905-14.
[10] WU J, Kobbelt TL. Structure recovery via hybrid variational surface approximation. J Computer Graphics Forum 2005;24:277-84.
[11] Julius D, Kraevoy V, Sheffer A. D-charts: Quasi-developable mesh segmentation. Computer Graphics Forum 2005;24:981-90.
[12] Castro PMN, Lima LAP, Lucena FLA. Invariances of single curved manifolds applied to mesh segmentation. J Computers &Graphics 2014;38:399-409.
[13] Wu Z, Shou R, Wang Y, Liu X. Interactive shape co-segmentation via label propagation, J Computers & Graphics 2014;38:248-54.
[14] Lloyd S. Least square quantization in PCM. IEEE Trans. Inform. Theory 1982;28:129-37.
[15] Son M, Lee Y, Kang H, Lee S. Structure grid for directional stippling. J Graphical Models 2011;73:74-87.
[16] Qin H, Wang Y, Li M, Zhou H. An accelerated scheme with high quality mesh based on Lloyd iteration. J Cent. South Univ 2012;19:2797-802.
[17] Rebollo-Monedero D, Forné J, Pallarès E, Parra-Arnau J. A modification of the Lloyd algorithm for k-anonymous quantization. J Information Sciences 2013;222:185-202.
[18] Ciarlet PG, Iserles A, Kohn RV, Wright MH. Geometry and Topology for Mesh Generation. 1th ed. New York: Cambridge University Press; 2001.
[19] Har-Peled S. A replacement for Voronoi diagrams of near linear size, in: Proc IEEE FOCS 2001;94–103.